

Holographic entanglement entropy beyond coherent states

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We study entanglement entropy for a class of states in quantum field theory that are entangled superpositions of coherent states with well-separated supports, analogous to Einstein-Podolsky-Rosen or Bell states. We find the contributions beyond the area law. In the case of strongly-coupled conformal field theories, we argue that these states are holographically dual to superpositions of bulk geometries. We conclude that for these states one can use the Ryu-Takayanagi holographic entanglement entropy formula to calculate some terms in the entanglement entropy, but that there can be additional $O(N^2)$ contributions. We argue that this class of states includes those generated by local quenches and thus that these cannot be described by a classical dual geometry. These considerations may be important for more fine grained treatments of holographic thermalization.

INTRODUCTION

The AdS/CFT correspondence [1] has been widely used to study certain strongly-coupled conformal field theories (CFTs) using the dual description of certain states in those theories as asymptotically anti-de Sitter (AdS) solutions of supergravity. One quantity of interest for CFT states is the entanglement entropy between some spatial region A and its complement, and there is a proposal for how to compute this from the dual geometry [2, 3] that has by now accumulated a large amount of evidence in its favor. This proposal implicitly assumes the existence of a dual geometry.

However, many states of the CFT will have duals that cannot be described geometrically. One purpose of this letter is to argue that the state generated by a local quench of a CFT is such a state. Local quenches are interesting dynamical processes that have been studied in detail in conformal field theory, e.g., [4–6]. There have been attempts to find the dual geometry to the state generated by a local quench, but none have so far succeeded, which is consistent with our claim that no such dual geometry exists.

Local quenches are important because they may offer a way to measure entanglement entropy [7] and because they offer a localized version of the dynamics responsible for thermalization after a global quench [8–10]. Such global quenches have the starting point for all extant studies of holographic thermalization, see, e.g., [3, 11–17]. In light of this, another purpose of this letter is to describe a general class of states similar to the local quench states and to investigate their entanglement entropy, to see how the proposal in [2] might be generalized.

DESCRIPTION OF STATES

For a given AdS/CFT duality, one expects that there is a large class of states of the CFT that are not dual to a classical bulk geometry, in particular states that are (sufficiently) non-coherent with respect to their field expectation values.¹ This

point has been made before, e.g., in [18, 19] in the context of the 1/2 BPS sector of the $\mathcal{N} = 4$ SYM theory. Without attempting to describe exactly which states are dual to a classical bulk geometry, let us refer to such states as coherent.

We can construct a simple example of a non-coherent state, reminiscent of the Einstein-Podolsky-Rosen thought-experiment [20], i.e., two well-separated bits of matter in an entangled state. This is also called a Bell state after the well-known work [21]. To set this up, divide the space of a quantum field theory into two regions. For simplicity, consider a one-dimensional space. Denote the regions left and right of the origin as A and B , respectively, then let us assume we can write the Hilbert space of the full theory as a tensor product $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$.

Now let ϕ be some quantum field. For simplicity suppose ϕ is a real scalar field. Then consider a state $|\phi_A^+\rangle$ of the field corresponding a lump of matter away from the origin. More specifically, consider a coherent state of the field operator with a positive Gaussian profile, i.e., $\langle \phi_A^+ | \phi(x) | \phi_A^+ \rangle = A_0 e^{-(x+a)^2/\sigma^2}$. Then we can also consider the state with a negative profile: $\langle \phi_A^- | \phi(x) | \phi_A^- \rangle = -A_0 e^{-(x+a)^2/\sigma^2}$. Similarly for region B we have $\langle \phi_B^\pm | \phi(x) | \phi_B^\pm \rangle = \pm A_0 e^{-(x-a)^2/\sigma^2}$. Well separated means $a \gg \sigma$. By taking A_0 sufficiently large we can ensure that $\langle \phi_{A,B}^\pm | \phi_{A,B}^\mp \rangle \approx 0$ and furthermore that $\langle \phi_{A,B}^\pm | O(x) | \phi_{A,B}^\mp \rangle \approx 0$ where $O(x)$ is the field operator, the momentum operator, or the Hamiltonian.²

Further consider the states $|\phi_A^s\rangle = \frac{1}{\sqrt{2}}(|\phi_A^+\rangle + |\phi_A^-\rangle)$ (and analogously defined $|\phi_B^s\rangle$). Here “s” stands for superposed. We assume that the Hamiltonian has a $\phi \rightarrow -\phi$ symmetry so all these states have the same energy.

¹ Mixed states of the CFT may have good geometric duals, e.g., black holes, but these are usually interpreted as coarse-grained dual descriptions. Here

I’m not considering coarse graining beyond tracing over the complement of a spatial region, so throughout this letter I will only be considering pure states of the CFT.

² We are assuming a conventional field theory that admits coherent states and has a Hamiltonian with terms polynomial in the field and momentum operators.

Now for the total state of the CFT we have options, e.g.,

$$\text{Entangled: } |\psi_1\rangle = \frac{1}{\sqrt{2}}(|\phi_A^+\rangle|\phi_B^-\rangle + |\phi_A^-\rangle|\phi_B^+\rangle) \quad (1)$$

$$\text{Unentangled: } |\psi_2\rangle = |\phi_A^s\rangle|\phi_B^s\rangle. \quad (2)$$

Both states have the same energy and, in fact, the same stress-tensor expectation value $\langle T_{\mu\nu}(x) \rangle$ and the same expectation values for the field, $\langle \phi(x) \rangle = 0$, for all x , for large A_0 . However, we would expect that state 1 has roughly $\log 2$ more entanglement entropy than state 2. Below we will verify this expectation in a simple model where we can explicitly compute the entanglement entropy, including area law contributions.

In a holographic context, we may consider a CFT that has a holographic dual, for example the canonical $\mathcal{N} = 4$, four-dimensional $SU(N)$ gauge theory. The large N , large coupling limit of the CFT is conjectured to be dual to type IIB supergravity on an asymptotically $AdS_5 \times S^5$ spacetime [1]. There is an analogous conjectured duality between certain two-dimensional CFTs and asymptotically AdS_3 spacetimes.

In the case of coherent states of the CFT that can be matched with a particular geometry, i.e., a solution to the appropriate supergravity theory, there is a proposal for how to calculate the entanglement entropy of the CFT state. For a spatial region A , the entanglement entropy S_A of the reduced density matrix ρ_A (traced over the complement of A) is given by

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}, \quad (3)$$

where γ_A is the extremal surface of minimal area in the bulk whose boundary coincides with the boundary of A [2, 3]. In the duality G_N^{-1} goes like N^2 , so in the large N limit the above formula will only capture $O(N^2)$ contributions.

In this context there are states analogous to those in (1) and (2) that we can obtain by superposing coherent traveling excitations in the CFT. States describing a single traveling excitation, a “pulse,” and their holographic duals were recently described in [22], in AdS_3/CFT_2 . The states in [22] were mixed states of the CFT, but there are corresponding pure states with the same essential properties, i.e., supported in a finite region of the null coordinates and with $O(N^2)$ of the fields excited. We can also consider two left-moving excitations that are separated into the left and right regions of the CFT for some period of time, see Fig. 1.

We can construct pure states corresponding to the pulses either being entangled or not, in analogy with (1) and (2) above. We expect the maximally entangled state, call it $|\psi_1\rangle$, will have $O(N^2)$ more entanglement entropy than the unentangled state $|\psi_2\rangle$. We will corroborate this expectation with an explicit calculation in a toy model below. Let us also note that the maximally entangled state is similar to that investigated in [6], which was produced after a local quench. The entanglement entropy was found there to be proportional to the central charge, which would be $O(N^2)$ in a holographic theory.

We can argue, by contradiction, that these states $|\psi_{1,2}\rangle$ do not have a dual bulk geometry. The first premise is that the

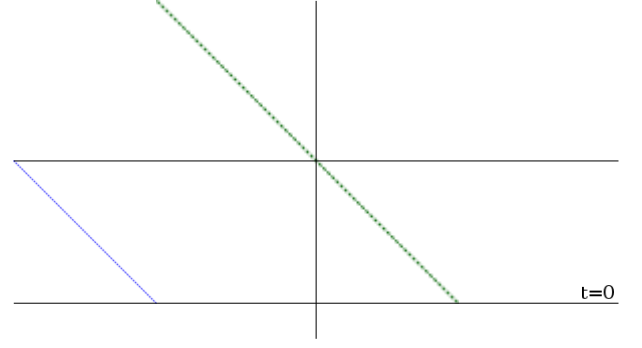


FIG. 1. Two left moving excitations in the CFT, which may be entangled or not.

entangled state will have $O(N^2)$ more entanglement entropy than the unentangled state. The second is that, assuming these two states had a dual bulk geometry, they would be same. The reason for this is that these two states are identical as far as $\langle T_{\mu\nu}(x) \rangle$ and the expectation values of the fields, to $O(N^2)$, so this is what we expect from the usual AdS/CFT dictionary [23, 24]. Then, using (3) we would find that the $O(N^2)$ entanglement entropy would be the same, a contradiction.

The point of the above argument is just to emphasize that simple superpositions of coherent states are in the class of states that don’t have a dual bulk geometry. Such simple superpositions are interesting for at least two reasons. First, while (3) is not directly applicable, it may still provide some terms in the full expression for the entanglement entropy of certain superpositions, as we discuss further in the Conclusion. Second, states of this kind are produced by quenches, as I will argue later, and may be important in general dynamical situations.

MODEL FOR CALCULATING ENTANGLEMENT ENTROPY

We can investigate some of these issues explicitly by considering a toy model of the ground state of a free quantum field theory. The point of this toy model is only to capture the leading contributions to the entanglement entropy. From early investigations of the area law [25, 26], the entanglement entropy of a region of space in the ground state of a field has been understood to come from correlations across the boundary of the region. This explains why the entropy goes as the area and why it diverges in the absence of a UV cutoff—arbitrarily small distances near the boundary can contribute.³

³ This explanation is only compelling in the case of fields whose correlation length is small compared to the dimension of the region, i.e., sufficiently massive fields. However, the area law is still gives the leading order behavior in the massless case [25] and so our model should still be relevant for that case.

We can model this situation by considering a discrete space. For now just take the radial direction to be discrete, indexed by i so $r = ia$ where a is the lattice spacing. Decompose the Hilbert space of the theory into a tensor product, one factor for each i . Then the entanglement between different points in the space can be roughly modeled with a state of the form

$$|0\rangle \approx \left(\sum_{i=1}^{i_{\max}-1} e^{a_i^\dagger a_{i+1}^\dagger} \right) \prod_{i=1}^{i_{\max}} |0\rangle_i, \quad (4)$$

where $i_{\max}a$ is some IR cutoff, not important for our purposes here. This exhibits entanglement between nearest neighbor radii. Tracing over the region insides a sphere of radius R corresponds to tracing over factors up to and including $i_R = R/a$ and the only contribution to the entanglement entropy would be from the term containing $\exp(a_{i_R}^\dagger a_{i_R+1}^\dagger) |0\rangle_{i_R} |0\rangle_{i_R+1}$. The trace over the i_R factor would result in a reduced density matrix for the $i_R + 1$ that was mixed and in fact thermal, i.e., of the form $\sum_\alpha e^{-\beta E_\alpha} |E_\alpha\rangle_{i_R+1} \langle E_\alpha|_{i_R+1}$. The corresponding entanglement entropy would thus be the von Neumann entropy of a thermal state on a sphere, which we expect to be extensive with respect to the area of the sphere, and so we get an area law: $S_{|0\rangle} \propto R^{d-1}$, if the dimension of space is d . The above model is clearly quite crude and only captures the leading, area-law behavior of the entanglement entropy. However this is the leading order behavior even in the vacuum of the strongly-coupled field theory [27].

Now let us consider an excited state which contains entanglement besides the nearest neighbor type considered above. We can model this with states

$$|\psi_\pm\rangle = C_1 e^{\pm \lambda a_1^\dagger \mp \lambda a_j^\dagger} |0\rangle \quad (5)$$

$$= \left(\sum_{i=1}^{i_{\max}-1} e^{a_i^\dagger a_{i+1}^\dagger} \right) |\pm\lambda\rangle_1 |0\rangle_2 \cdots |\mp\lambda\rangle_j \cdots, \quad (6)$$

where $\lambda > 0$ and $j \gg 1$. The $|\pm\lambda\rangle$ are coherent states. These are crude models for the $|\psi^\pm\rangle$ considered above. Each of the states $|\psi_\pm\rangle$ are unchanged from the vacuum in terms of their entanglement entropy: they will each obey an area law. We now want to consider a superposition of these coherent states

$$|\psi\rangle = C_1 |\psi_+\rangle + C_2 |\psi_-\rangle, \quad (7)$$

where for simplicity assume that the coefficients C_i are real, and a trace over radii smaller than ja . The resulting reduced density matrix is

$$\begin{aligned} \rho_{\text{red}} = & C_1^2 \left(\sum_\alpha e^{-\beta E_\alpha} |E_\alpha\rangle_{i_R+1} \langle E_\alpha|_{i_R+1} \right) \cdots |-\lambda\rangle_j \langle -\lambda|_j \cdots \\ & + C_2^2 \left(\sum_\alpha e^{-\beta E_\alpha} |E_\alpha\rangle_{i_R+1} \langle E_\alpha|_{i_R+1} \right) \cdots |+\lambda\rangle_j \langle +\lambda|_j \cdots \end{aligned} \quad (8)$$

Luckily this form is diagonal (choosing a coherent state basis for the j th factor), and we can compute the entanglement

entropy. We get

$$S_{|\psi\rangle} = S_{|0\rangle} - \gamma \log \gamma - (1 - \gamma) \log(1 - \gamma), \quad (9)$$

where $\gamma = C_1^2$. For small γ this is

$$S_{|\psi\rangle} \approx S_{|0\rangle} - \gamma \log \gamma. \quad (10)$$

Of course, if we had chosen an entangling region such that $R > ja$, then the entanglement entropy would return to its vacuum value.

We can generalize this to some number M fields ϕ_k . Denote the coherent state of field k at radial position i by $|\lambda\rangle_{k,i}$. Now we imagine that all of these fields are excited near the origin and at radial position ja . One example of such a state is

$$\begin{aligned} |\psi_{(\pm \dots \pm)}\rangle = & |\pm\lambda\rangle_{1,1} |0\rangle_{1,2} \cdots |\mp\lambda\rangle_{1,j} \cdots \\ & \otimes |\pm\lambda\rangle_{2,1} |0\rangle_{2,2} \cdots |\mp\lambda\rangle_{2,j} \cdots \\ & \cdots \\ & \otimes |\pm\lambda\rangle_{M,1} |0\rangle_{M,2} \cdots |\mp\lambda\rangle_{M,j} \cdots, \end{aligned} \quad (11)$$

where we have suppressed the nearest-radial-neighbor entanglement operators for brevity. The index $\beta = (\pm \dots \pm)$ runs over the 2^M possible states. The maximal entropy such state is

$$|\psi\rangle = \frac{1}{\sqrt{2^M}} \sum_\beta |\psi_{(\pm \dots \pm)}\rangle. \quad (12)$$

If we again trace over radii smaller than ja we find an entanglement entropy

$$S_{|\psi\rangle} = S_{|0\rangle} + M \log 2. \quad (13)$$

It's also easy to construct an unentangled superposition by analogy with (2), by using states $|\psi^s\rangle_{1,k} = 2^{-1/2}(|+\lambda\rangle_{1,k} + |-\lambda\rangle_{1,k})$. This will share with the state in (12) the property that $\langle \phi_k(x) \rangle = 0$ for each field, but will have entanglement entropy equal to that of the vacuum.

DISCUSSION

We now consider the implications for the holographic entanglement entropy proposal. Can we compute the entanglement entropy for such non-coherent states of the boundary CFT, of the kind discussed above, from bulk information? The simple model considered above suggests that the leading order entanglement entropy for the simplest such state in the CFT is given by

$$S_A \approx \frac{\text{Area}(\gamma_A)}{4G_N} + N^2 C \log 2, \quad (14)$$

where C is a constant of order 1, the fraction of the $O(N^2)$ fields that are entangled. This does not tell us how to compute

the entanglement entropy from the bulk, rather it is a constraint on any proposal for computing S_A from bulk data for a states of this kind. However, from the bulk point of view, the state might be interpreted as a superposition of geometries.

We can consider a slightly more general situation. First, if the state of your boundary CFT is in a (sufficiently) coherent state, then you can use the original (3). If not, write your state in a basis of coherent states. For simplicity let us consider the simplest case of a state that describes $O(N^2)$ fields supported in two regions that are well separated and in the maximally entangled state. If your entangling surface separates those regions, then the model suggests the conjecture that the entanglement entropy is given by (14), to leading order in N , where $\text{Area}(\gamma_A)$ is computed in pure AdS.

This all refers to the special case of superpositions of localized coherent states. We expect the bulk geometries dual to these various coherent states to be the same throughout most of the spacetime and this is why (3) might still be used to compute a term in the full expression for the entanglement entropy. A general, non-coherent state may be dual to a superposition of wildly different bulk geometries and wildly different field configurations and it would be interesting to find generalizations of (14) for such states, if such exist.

The additional term appearing in (13) can be identified with the entanglement entropy produced following a local quench in a general CFT. Though it is difficult to directly access the state of the CFT after the local quench (although see [28, 29]), studies of entanglement entropy after a local quench [4–6] suggest that the state consists of entangled coherent particles emitted from the quench point(s) and then propagating at the speed of light. The quench thus creates a state much like that in (12). Such a quench, which corresponds to a topology change, is generally expected to excite every field in the theory [30]. This is corroborated by the fact that the entanglement entropy produced by a local quench in 2D CFTs is proportional to the central charge [4–6]. In the holographic context this would mean $O(N^2)$ contributions to the entanglement entropy of the form (14). Thus, we do not expect there to be any classical bulk geometry dual to a local quench of the boundary CFT.

These considerations may be important for holographic thermalization, see, e.g., [3, 11–17]. Like local quenches, global quenches in $1+1$ dimensions are known to produce pairs of entangled particles that can become separated by arbitrarily large distances [8–10]. This picture seems to hold at strong coupling and in higher dimensions as well, at least for the entanglement entropy of a single region [16]. The spatially homogenous nature of the global quench means that one will not get non-coherent states of the kind considered here, and the classical dual geometries commonly used (such as the Vaidya metric) seem to offer accurate coarse grained descriptions. However, more general thermalization scenarios may lead to very non-coherent states, and so modeling such a process by a single dual geometry may not be correct, even to leading order in N , since the resulting calculations of entanglement entropy may be missing terms such as those appear-

ing in (14). We hope to study this in more detail in future work.

We note that there has already been a holographic study of an entangled pair of spatially separated subsystems, in [31], Sec. 7.1. In that system the entanglement in the vacuum state could be understood as an Unruh effect due to the fact that the CFT was undergoing uniform acceleration.

We acknowledge that the model we presented is crude and there are many avenues for further investigation. It would be nice to improve the model so as to capture the subleading corrections (in R/a) to the area law for states of the kind we are considering, to see how they compare. We should also investigate the mutual information and the tripartite information, since these show departures from the entangled particle picture of thermalization after a global quench in the strongly coupled case [32]. We have been vague about how sufficiently coherent a state needs to be in order to apply the holographic entanglement entropy proposal. It would be interesting to quantify this and study departure from coherence in a dynamical situation and the resulting behavior of the entanglement entropy and other quantities.

I thank S. Avery, V. Balasubramanian, D. Berenstein, B. Craps, T. Hertog, D. Marolf, E. Perkins, M. Roberts, V. Rosenhaus, M. Srednicki and M. Van Raamsdonk for all their helpful conversations about this work, and W. Kelly, M. Headrick, G. Horowitz and T. Takayanagi additionally for their comments on an early draft. The author was supported in part by the Department of Energy under Contract DE-FG02-91ER40618, and part of this work was carried out while participating in the KITP program “Bits, branes, and black holes” and the associated conference “Black holes and information” and was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915.

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